KNOWLEDGEABLE RESEARCH (KR)
ISSN: 2583-6633 | CODEN: KRABAU
International Open-Access Peer-Reviewed Refereed Multidisciplinary Journal
https://knowledgeableresearch.com
Vol. 5, Issue 08,2026

Received: 05/07/2026 | Accepted: 22/08/2026 | Published: 30/08/2026

Estimation of Satellite Thermal Network Model Parameters Based on Periodic Solution

Han Ju Gwon, *Kim Jin Hyok

[1]Faculty of Mathematics, **Kim Il Sung** University, Pyongyang, DPR Korea

Kim Kyong Ho[2], Sin Hyok In[2], Ri Jin Su[2], Kim Chol Jin[2]

[2] Faculty of Mechanical Engineering, Kim Chaek University of Technology, Pyongyang, DPR Korea

Abstract

Thermal network models are widely used in thermal analysis and design of satellites. A thermal control system is designed to maintain all components within feasible temperature ranges, when a satellite is often considered to be subjected to periodic thermal loads. Therefore, a thermal model should fully represent periodic temperature variations in steady stage. This paper proposes a new approach to determine parameters of a reduced model by using the periodic solution of a detailed model. In order to get the input data for the parameter estimation, the periodic solution of thermal network equations is approximately calculated with low computational costs. A genetic algorithm (GA) and a sequential quadratic programming (SQP) are adopted to enhance both global and local search capabilities in the parameter estimation. Several constraints are introduced to improve the behavior of the reduced model and to reduce the dimension of search space. Thermal analysis and estimation of thermal network parameters of a microsatellite are conducted to verify the methods.

Keywords: *Satellite, Thermal network, Inverse method, Parameter estimation, Periodic solution.*

***Corresponding Author:**
Kim Jin Hyok
Email: JH.KIM@star-co.net.kp

1.Introduction

Thermal network is a lumped parameter modeling approach, which is widely used to describe thermal dynamics of satellites. To find temperature variations, a higher order model (detailed model) is usually taken in thermal analysis, meanwhile a relatively simple model (reduced model) is used in thermal design which contains a large number of iterative calculations.

There are two categories of methods to get thermal network equations: direct and inverse methods. In the direct method, the distributed parameter system derived from the structure of a satellite and properties of materials is discretized [1]. Since the discretization usually gives a detailed model, a model reduction is needed to get a reduced thermal network. During the model reduction, a reduced structure and new parameters are obtained together but sometimes parameters should be corrected to conform the behavior of the reduced model to the detailed one in frequency and time domains [2, 3]. An inverse method uses analysis results of a detailed model, ground test data or flight measurements. The graph of a thermal network is drawn from the structure of a satellite and model parameters are estimated by various optimization methods. The inverse method is implicit discretization and model reduction, furthermore it includes the linearization of disregarded radiative couplings. In fact, it refers to system identification, the procedure of which consists of 4 steps: data acquisition, model order/structure selection, parameter estimation and model validation. This paper focuses on the data acquisition and the parameter estimation. Here a detailed model is analyzed to get input data instead of testing a real plant.

The goal of the system identification is to reduce errors between temperatures calculated by the identified model and the result of tests or analyses. Therefore, during the parameter estimation, solving of thermal network equations is repeated so that computational cost increases. The more the compared data, the more the computational cost, on the contrary, the smaller the amount of data, the lower the accuracy of identified model. Therefore, it is very important to select input data properly when estimating parameters of the reduced model by using analysis results of the detailed model.

Estimated parameters are closely related to the frequency characteristic and the time history of input data. However, the use of proper data has partially been dealt in thermal network identification. The identified or reduced model is unable to simulate the whole behavior of a real system under arbitrary external loads. As a rule, the most important and general behavior is simulated by using as simple model as possible, which is the steady temperature oscillation in the case of satellites. This requires a mathematical study on the dynamics of thermal model. Many authors have discussed steady and transient solutions of thermal network equations under various thermal loads. In particular, lots of qualitative and quantitative researches have been concerned with the existence and the stability of the periodic solution of thermal network equations under periodic thermal loads [4, 5, 6].

A thermal network model is expressed in terms of ordinary differential equations, the periodic solution of which can be obtained by several methods. In general, the direct integration of differential equations yields a large amount of calculation. Also, this question can be referred to a boundary value problem, however in a multi-nodal model, the calculation complexity increases enormously. The other way to use the Fourier transform inevitably requires linearization [7].

There are many approaches to estimate parameters of a satellite thermal model. Some of them use test and analysis data to correct parameters of the thermal network obtained by the direct method, in which parameter values are already given with a certain accuracy. They usually use local optimizations such as least square method [8], Kalman filter [9] and gradient-based method [10]. Then, both the computational cost and the accuracy are improved, however, a local convergence could take place. Others don' t need the initial estimation of parameters and usually refer to global optimizations such as GA [11, 12] and particle swarm optimization [13]. GAs have high ability to search global optimal solution but they have low convergence rate around an optimal point to increase computation time. In [14], The performance of GAs and gradient methods are compared from different viewpoints.

In this work, thermal network parameters are estimated by reference to the steady periodic solution, which is very important in thermal analysis and design of satellites.

The proposed method allows to get the approximate periodic solution by directly integrating thermal network equations and to use it as the input data for the parameter estimation. The periodic solution can be rapidly obtained by choosing steady state solution under averaged loads as an initial condition for the integration.

In the parameter estimation, the proposed method uses several constraints for the identified model to simulate both transient and steady state level of the long-term trend as well as the periodic part of temperature variation. At the beginning of the estimation, the global optimization is done by a GA and in the local optimization, an SQP is adopted to accelerate the convergence. The linear equality constraints mentioned above are also used to improve search efficiency. Finally, the payload compartment of an Earth observation microsatellite has been analyzed and modeled to verify these methods.

INPUT DATA

In this work, temperatures obtained by a detailed model are used as the input data to estimate parameters of a reduced model.

The lack information in the data set might result in exclusive parameters which fit only the data used in the estimation. The best way is to use the whole range of data including both steady and transient stage solutions, however, it increases computational cost. In the thermal design of a satellite, if thermal loads are periodic, we are interested only in the temperature variations in steady stage.

Therefore, the proposed approach estimates thermal network parameters by using the steady periodic solution as the input data.

1. Approximate calculation of periodic solution

The thermal network of a spacecraft is expressed as

$$C_i \frac{dT_i}{dt} = \sum_{\substack{j=1 \\ j\neq i}}^{n} K_{ij}\left(T_j - T_i\right) + \sum_{\substack{j=1 \\ j\neq i}}^{n} R_{ij}\left(T_j^4 - T_i^4\right) - R_{i\infty}T_i^4 + Q_i \qquad (1)$$

$$i = 1, \cdots, n$$

, where n is the number of nodes, T_i is the temperature of a node, C_i is the nodal thermal capacity, K_{ij} and R_{ij} are linear and radiative conductances between nodes, $R_{i\infty}$ is the radiative coefficient of a node to the outer space and Q_i is the thermal load acting on a node.

Thermal loads arise from heat dissipation of devices and environmental heating fluxes. In many cases including Sun-synchronous satellites, disregarding the annual motion of the Earth and variations of orbital parameters, and considering only the basic regime of attitude control, we can notice that heating fluxes vary with the same period as the revolution of a satellite. Heat dissipation of devices depends on the mission of a satellite, which is closely related to the orbit and attitude. As a result, it also changes with a certain period, which is closely related to the orbital period in many cases. Therefore, in thermal analysis and design of satellites, periodic thermal loads are usually considered.

Denoting by t_p the period of thermal load, we can write

$$Q_i(t) = Q_i(t + t_p) \tag{2}$$

The behavior of thermal network model has two stages: transient stage significantly affected by an initial condition and steady stage related only to thermal loads. In particular, the trajectory of temperatures under periodic thermal loads consists of long-term trend and oscillation around it. By linearizing nonlinear terms in a sense, the average behavior of the solution under periodic loads can be approximated by the solution under average loads in both transient and steady stages.

The steady periodic solution of a detailed model should be calculated to acquire the input data for the parameter estimation of a reduced model. In this paper, the approximate periodic solution is calculated in a simple way. The long-term behavior of thermal network under periodic loads corresponds to a process that the thermal energy level of the whole system gets balanced with space environment. Therefore, the average of periodic solution can be approximated by the steady state solution obtained under average thermal loads. If we use that solution as an initial condition, we can obtain the periodic solution by integrating thermal network equations until the oscillation of temperatures is fully developed.

Steady state analysis of detailed model

The steady state solution of thermal network equations given by Eq. (1) can be obtained by

$$\sum_{\substack{j=1\\ j\neq i}}^{n} K_{ij}(\bar{T}_j - \bar{T}_i) + \sum_{\substack{j=1\\ j\neq i}}^{n} R_{ij}(\bar{T}_j^4 - \bar{T}_i^4) - R_{i\infty}\bar{T}_i^4 + \bar{Q}_i = 0 \qquad i = 1, \cdots, n \tag{3}$$

, where $\bar{Q}_i$s are the average thermal loads over a period.

$$\bar{Q}_i = \frac{1}{t_p}\int_t^{t+t_p} Q_i(\tau)d\tau \tag{4}$$

The existence and the uniqueness of the solution of nonlinear algebraic equations in Eq. (3) have been proved in [4] and there are many solution methods [15, 16].

We use the solution of Eq. (3) as the initial condition to integrate Eq. (1). If that solution is a point on the trajectory of a periodic solution at a certain time, the periodic solution is completely obtained by integrating over only one period from that time. In general, it is impossible that all $\bar{T}_i$s respectively belong to the corresponding solution trajectories at each node at the same time, however we can find the time at which the energy of the satellite calculated from $\bar{T}_i$ s belongs to the energy trajectory corresponding to the periodic solution. In order to determine that time approximately, the one-nodal analysis is carried out to get the energy trajectory of a satellite.

One-nodal analysis

Regarding the whole satellite as one node, we obtain

$$C\frac{dT}{dt} = -R_\infty T^4 + Q \tag{5}$$

,

where T is the average temperature of the satellite. C is the whole capacity and Q is the sum of thermal loads, which have following relations with those in Eq. (1):

$$C = \sum_{i=1}^{n} C_i \tag{6}$$

$$Q = \sum_{i=1}^{n} Q_i \tag{7}$$

R_∞ is the radiative coefficient from the satellite to the outer space, which can be approximated as

$$R_\infty \approx \sum_{i=1}^{n} R_{i\infty} \tag{8}$$

In practice, it might be more convenient to directly calculate C, R_∞ and Q.

Let us suppose that the variation of T is close enough to the steady periodic solution after integrating Eq. (5) over $r+1$ periods under the periodic load Q. In the steady periodic solution, the whole energy of the satellite CT varies periodically. Thus, it is possible to find a point on the energy trajectory which has the same energy as the one calculated from $\bar{T}_i$s.

$$CT\left(rt_p + t_d\right) = \sum_{i=1}^{n} C_i\bar{T}_i \tag{9}$$

There exist at least two values of t_d that meet Eq. (9) and $0 \le t_d < t_p$ within one period. If the phase of Q_is is brought forward by t_d and $\sum_{i=1}^{n} C_i\bar{T}_i$ is selected as the initial condition, the periodic energy trajectory is completely determined by integrating Eq. (5) over one period.

Transient analysis of detailed model

If the phase of Q_i is brought forward by t_d and $\bar{T}_i$ is selected as an initial condition of Eq. (1), it can be considered that temperatures have converged to the periodic solution in the sense of the energy. Replacing the thermal loads in Eq. (1) with $Q_i'(t) = Q_i(t + t_d)$, we can write

$$C_i \frac{dT_i}{dt} = \sum_{\substack{j=1 \\ j \neq i}}^{n} K_{ij}\left(T_j - T_i\right) + \sum_{\substack{j=1 \\ j \neq i}}^{n} R_{ij}\left(T_j^4 - T_i^4\right) - R_{i\infty} T_i^4 + Q_i' \tag{10}$$

$$T_i(t_0) = \bar{T}_i, \qquad i = 1, \cdots, n$$

Therefore, the approximate periodic solution is obtained by integrating Eq. (1) over only a few periods until the oscillation is fully developed.

Preparation of input data

A reduced thermal network model is given by

$$C_{ri} \frac{dT_{ri}}{dt} = \sum_{\substack{j=1 \\ j \neq i}}^{n_r} K_{rij}\left(T_{rj} - T_{ri}\right) + \sum_{\substack{j=1 \\ j \neq i}}^{n_r} R_{rij}\left(T_{rj}^4 - T_{ri}^4\right) - R_{ri\infty} T_{ri}^4 + Q_{ri}' \tag{11}$$

$$i = 1, \cdots, n_r, \qquad n_r < n$$

, where the subscript r denotes the reduced model.

The nodes of the reduced thermal network should be selected among nodes of the detailed thermal network to grasp the main temperature distribution of a satellite. The number and position of nodes can be controlled according to the task of the reduced model. In general, one or two nodes are assigned to each device, then thermal loads on each node Q_{ri}'s can be calculated according to the specific situation.

From the periodic solution of the detailed model obtained by solving Eq. (10), we extract the sufficient samples $T_{ri}(t_0), T_{ri}(t_1), \cdots, T_{ri}(t_N)$ corresponding to the nodes of the reduced model to use as the input data for the parameter estimation. From Eq. (10), it can be noticed that $T_{ri}(t_0) = \bar{T}_{ri}$.

PARAMETER ESTIMATION

Objective function

In many cases, $R_{ri\infty}$ is easily computed so C_{ri}, K_{rij} and R_{rij} are considered to be unknown parameters, which have the following relations:

$$K_{rij} = K_{rji}, \quad R_{rij} = R_{rji} \tag{12}$$

The number of nodal capacities of the reduced model is the same as the number of nodes n_r.

Considering Eq. (12), both the number of linear conductances n_{rk} and the number of radiative conductances n_{rr} are smaller than $n_r(n_r - 1)/2$. In practice, they are a few times as large as n_r, because conductive and radiative couplings don’t exist between every node.

Let us denote the vector which consists of unknown parameters by $\boldsymbol{\theta}_r$.

The objective function is defined in terms of the weighted mean square error (WMSE) as

$$J = \left\{ \frac{1}{N} \sum_{k=1}^{N} \sum_{i=1}^{n_r} w_{i0} \left[T_{ri}(t_k) - \hat{T}_{ri}(t_k, \boldsymbol{\theta}_r) \right]^2 \right\}^{\frac{1}{2}} \tag{13}$$

,
where $\hat{T}_{ri}(t_k, \boldsymbol{\theta}_r)$ is the solution of Eq. (11)
The w_{i0} s are weight factors to relatively consider temperature variations which have different amplitudes on each node during the steady oscillation. They are defined as

$$w_{i0} = \frac{w_i}{\sum_{i=1}^{n_r} w_i} \tag{14}$$

$$w_i = \left[\frac{1}{\max\limits_k T_{ri}(t_k) - \min\limits_k T_{ri}(t_k)} \right]^2 \tag{15}$$

Constraints

The upper and lower bounds of parameters can be determined from physical meanings and approximate calculations.

$$\theta_{ril} \le \theta_{ri} \le \theta_{riu}, \qquad i = 1, \cdots, n_r + n_{rk} + n_{rr} \tag{16}$$

When only the steady periodic solution is used as the input data, the estimated temperatures $\hat{T}_{ri}(t_k, \boldsymbol{\theta}_r)$ have a small difference with $T_{ri}(t_k)$ in the interval $[t_0, t_N]$, but if the interval is extended, the difference become larger so that the average energy level of the periodic solution could shift to the different level. To overcome this undesired problem, considering that the average of the periodic solution can be approximated by the steady state solution under the average loads, we introduce new equality constraints as follows:

$$\sum_{\substack{j=1 \\ j \ne i}}^{n_r} \left(\bar{T}_{rj} - \bar{T}_{ri} \right) K_{rij} + \sum_{\substack{j=1 \\ j \ne i}}^{n_r} \left(\bar{T}_{rj}^4 - \bar{T}_{ri}^4 \right) R_{rij} = R_{ri\infty} \bar{T}_{ri}^4 - \bar{Q}'_{ri} \tag{17}$$
$$i = 1, \cdots, n_r$$

, where $\bar{Q}'_{ri}$s are the average of thermal loads exerted on the reduced model.

$$\bar{Q}'_{ri} = \frac{1}{t_p} \int_t^{t+t_p} Q'_{ri}(\tau) d\tau \tag{18}$$

Eq. (17) is the linear equality constraint because it is linear with respect to the thermal parameters, though it is nonlinear with respect to the temperature. Taking the sum of all equations in Eq. (17) over index sets $i = 1, \cdots, n_r$, both sides become zero. This means that equations in Eq. (17) are linearly dependent, furthermore, the number of independent equations is $n_r - 1$.

Another problem in the parameter estimation is that the reduced model does not match with the detailed model in the transient behavior. In fact, the transient stage is the process during which the total thermal energy of the satellite become settled and dynamically balanced with the outer space. The variation of the total energy can be simulated by the one-nodal analysis in Eq. (5). Therefore, once the radiative coefficient from the satellite to the space is given, it is related only to the thermal capacity.

Since the total capacity of the satellite is easy to calculate, we impose the equality constraint to the sum of nodal capacities of the reduced model given by

$$\sum_{i=1}^{n_r} C_{ri} = C \tag{19}$$

After all, the optimization problem is to determine unknown parameters in $\boldsymbol{\theta}_r$ that minimize the objective function in Eq. (13) including temperatures calculated by Eq. (11) under the constraints in Eqs. (16), (17) and (19).

Optimization algorithm

As mentioned above, when estimating parameters of the reduced model by the inverse method instead of the direct reduction of the detailed model, we can know only the range of parameters at the beginning. In addition, the estimation of thermal model parameters is a typical multimodal problem, which usually needs global search algorithms. GA is widely used for the global optimization to estimate thermal model parameters [7, 9].

GAs provide excellent global search capability, but often have a low convergence rate in the local search. Therefore, we proposed an approach which uses the GA at the beginning and the SQP at the end.

Global search

There are a lot of unknown parameters and the objective function is continuous with respect to them, thus we use a real-coded genetic algorithm (RCGA). In the RCGA, the variables appear directly in the chromosome, so encoding and decoding are unnecessary. Roulette wheel selection, arithmetic crossover and Gaussian mutation are included in a set of GA operations and the elitist strategy is used.

The range of parameters and equality constraints are given. Since the equality constraints are linear, the dimension of the search space of the GA can be effectively reduced using them. Reducing the number of unknown capacities to $n_r - 1$, we determine one capacity by Eq. (19). Similarly, reducing the number of unknown linear conductances to $n_{rk} - n_r + 1$, we obtain the rest $n_r - 1$ conductances by Eq. (17). As a result, the dimension of the search space is reduced to $n_{rk} + n_{rr}$.

Without loss of generality, we assume that $\theta_{ri}(i = 1, \cdots, n_{rk} + n_{rr})$ are directly searched as optimization variables of the RCGA and the others, $\theta_{ri}(i = n_{rk} + n_{rr} + 1, \cdots, n_r + n_{rk} + n_{rr})$ are calculated by linear equality constraints. Then constraints in Eq. (16) to the range of parameters can be divided two groups as follows:

$$\theta_{ril} \leq \theta_{ri} \leq \theta_{riu}, \qquad i = 1, \cdots, n_{rk} + n_{rr} \tag{20}$$

$$\theta_{ril} \leq \theta_{ri} \leq \theta_{riu}, \qquad i = n_{rk} + n_{rr} + 1, \cdots, n_r + n_{rk} + n_{rr} \tag{21}$$

In the RCGA, the constraints in Eq. (20) can be simply satisfied in such a way that all individuals are forced to be within the search space throughout generation of an initial population, crossover and mutation.

Unlike them, the constraints in Eq. (21) are implemented by a penalty function method. A penalty function is defined as

$$P = J + \rho \sum_{i=n_{rk}+n_{rr}+1}^{n_r+n_{rk}+n_{rr}} [(min\{0, \theta_{riu} - \theta_{ri}\})^2 + (min\{0, \theta_{ri} - \theta_{ril}\})^2] \quad (22)$$

At the beginning, the GA is adopted to estimate parameters. If the decrease rate of the MSE becomes lower than a small threshold, the GA procedure is terminated and the local search begins.

Local search

We use the SQP as a local search algorithm [17]. The SQP has three stages: updating the Hessian of the Lagrangian function by BFGS method, finding of a new search direction by quadratic programming and determination of a step length by line search. It guarantees the super linear convergence. The equality constraints are implemented by the Lagrangian multiplier method.

RESULTS AND DISSCUSSION

Satellite configuration

The reduced thermal network model of a microsatellite is obtained by using the periodic solution of a detailed model. The satellite moves in the circular Sun-synchronous orbit with the inclination of 98.13° and the semi-major axis of 7064.7km. The primary attitude regime is Sun-pointing, but around 40° north latitude to accomplish the mission, Earth-pointing is kept for about 10 minutes. The orbital period is 5909s, which is considered to be the period of thermal load t_p in this case.

The satellite consists of three compartments and there exist only weak conductions through frames between them. Therefore, thermal models of compartments are separately built. In the paper, thermal modeling of the payload compartment is only discussed. Figure 1 shows the configuration of the payload compartment.

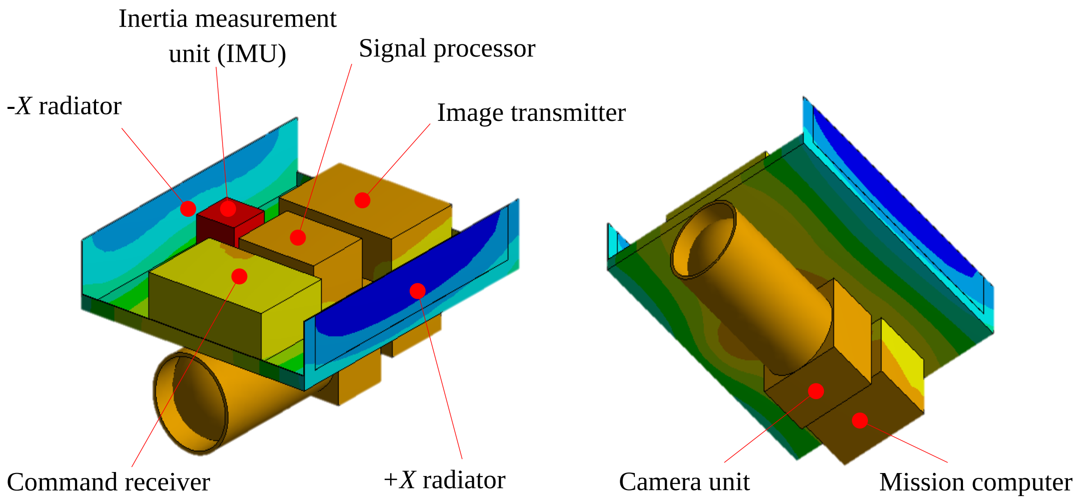

Figure 1. Thermal structure and steady state analysis result of payload compartment

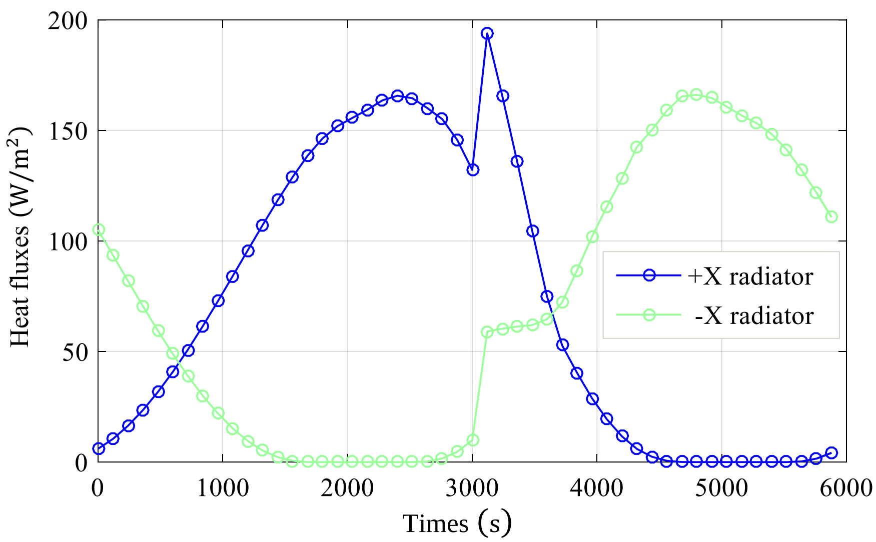

Figure 2. Heating fluxes absorbed by radiators

Figure 2 shows the heating fluxes absorbed at radiators during one period, which is calculated with the solar absorptivity $\alpha = 0.2$ and the infrared emissivity $\varepsilon = 0.8$. It is assumed that the Earth-pointing occurs from 3000s to 3600s and the attitude maneuver is neglected.

Heat dissipations, thermal capacities and operation modes of devices are shown in Table 1. There are two operation modes: intermittent and consistent. In the former the devices work only during Earth-pointing periods, while they always work in the latter. In both cases, heat dissipations of devices are assumed to be constant when they are active. The sum of thermal capacities of structures which are not shown in Table 1 is 359.6J/K, therefore, the total capacity of the payload compartment is $C = 4717.5\mathrm{J/K}$.

Table 1. Heat dissipation of payload devices

No	Device	Thermal capacity (J/K)	Heat dissipation (W)	Operation mode
1	Command receiver	700.4	7	Intermittent
2	Signal processor	383.8	5	Intermittent
3	Inertia measurement unit (IMU)	119.8	1.25	Consistent
4	Image transmitter	739.4	7	Intermittent
5	Camera unit	1666.7	1.4	Intermittent
6	Mission computer	737.0	6.1	Intermittent
7	$+X$ radiator	5.5	-	-
8	$-X$ radiator	5.5	-	-

Input data and model structure

Arranging nodes of reduced thermal networks on the places denoted by small red circles, we obtain the 8-node model. Nodes are numbered just the same as in Table 1. All results of the detailed model are discussed on the nodes corresponding to the reduced model from here.

First, the steady state analysis of a detailed model is done under the average thermal loads. The results are shown in Figure 1 and Table 2. Then, the number of nodes of the detailed model is $n = 1746$. The thermal energy of the whole satellite corresponding to the steady state is $\sum_{i=1}^{1746} C_i \bar{T}_i = 1.31526 \times 10^6 \mathrm{J}$.

Table 2. Steady state temperature under average loads

No	Devices	Temperature (℃)
1	Command receiver	6.302
2	Signal processor	6.516
3	Inertia measurement unit (IMU)	7.971
4	Image transmitter	6.029
5	Camera unit	6.337
6	Mission computer	6.095
7	$+X$ radiator	0.217
8	$-X$ radiator	1.883

The areas of $+X$ and $-X$ radiators are respectively $A = 0.012\mathrm{m}^2$, so the radiative coefficients to the space are $R = \varepsilon\sigma A = 5.5 \times 10^{-10}\,\mathrm{W/K^4}$. Therefore, the total radiative coefficient of the satellite is $2R = 1.1 \times 10^{-9}\,\mathrm{W/K^4}$. Based on these values, the one-nodal analysis is carried out over 50 periods. The obtained energy trajectory is shown in Figure 3.

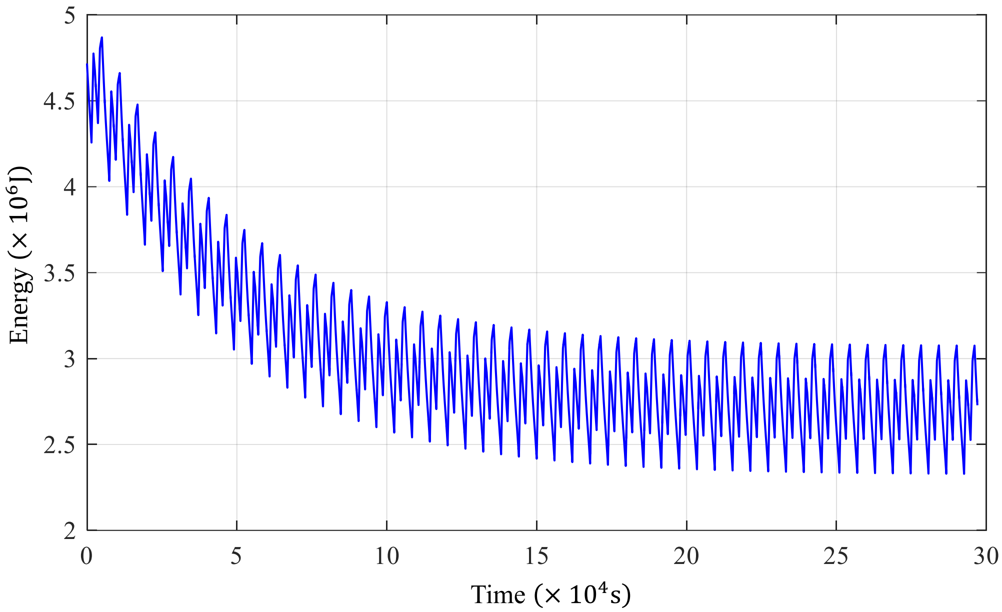

Figure 3. Energy trajectory obtained by one-nodal analysis

Figure 4 shows the process to determine a proper phase of thermal loads by using the data of the 50th period when the energy trajectory nearly converged to the periodic state. The dotted red line describes the energy level corresponding to the steady state under the average loads. There are four points where the red and blue lines cross each other, which depict the solutions of Eq. (9).

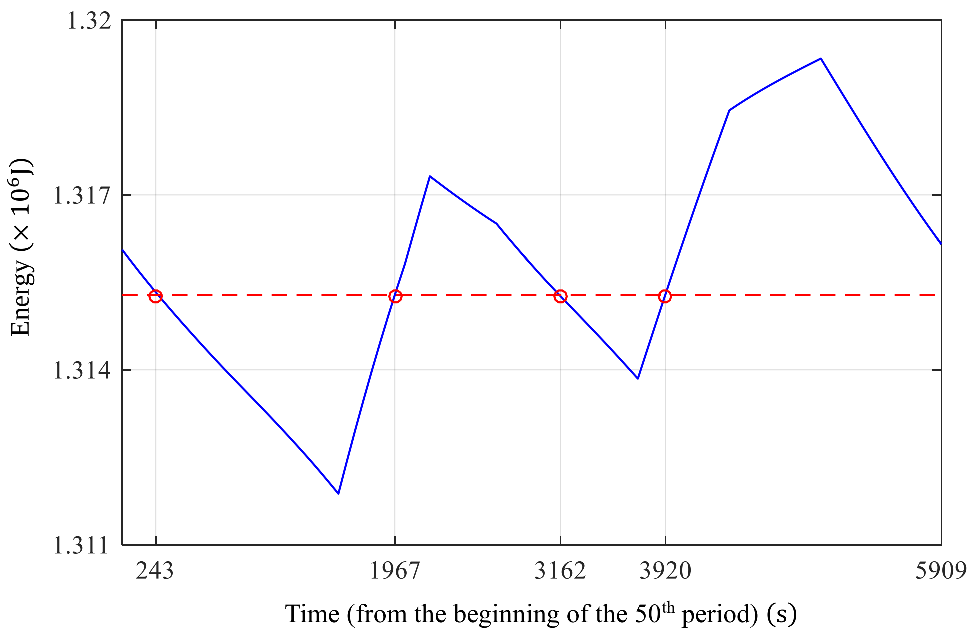

Figure 4. Determination of suitable phase of thermal loads

Next, the phase of loads is changed by selecting one of $t_d = 243s$, $1967s$, $3162s$, $3920s$ and the transient analysis is carried out over 30 periods by using the steady state result as an initial condition. Figure 5 shows the solution on nodes 1 and 7 which is obtained without the phase regulation of thermal loads. Then, moving the phase of loads forward by $t_d = 1967s$ according to the solution of Eqs. 5 and 9, another transient analysis is done, the result of which is shown in Figure 6. It can be seen that after regulating the phase of loads according to the one-nodal analysis result, the analysis over only 1~2 periods yields the result close enough to the periodic solution.

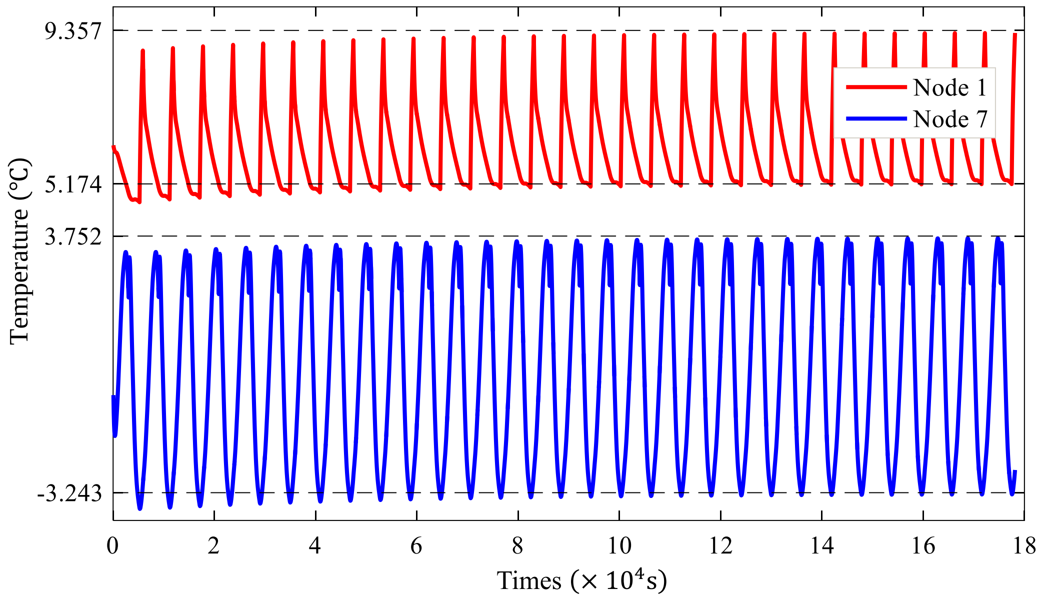

Figure 5. Transient solution of detailed model without phase regulation

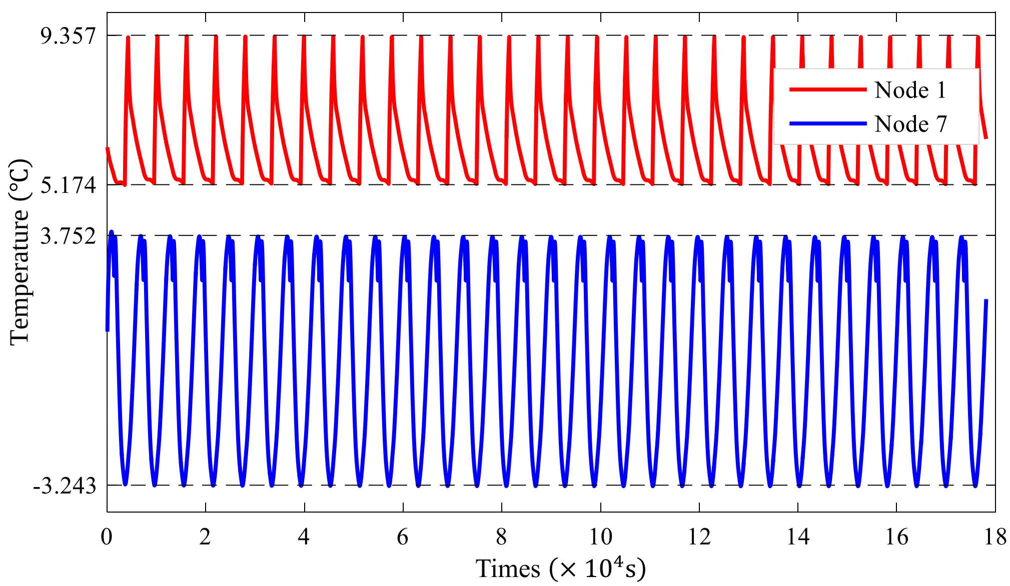

Figure 6. Transient solution of detailed model with phase regulation

Finally, for the parameter estimation, the data during 0~7200s are selected from the solution obtained by the phase regulation, because they properly represent the approximate periodic solution.

Considering the structure of the satellite and the analysis result of the detailed model, we set conductive and radiative couplings between nodes shown by the connected graph in Figure 7. The conductive couplings are set between nodes corresponding to adjacent devices. The radiative couplings are set between nodes which can "see" one another. Then, the radiations between nodes with small temperature differences are disregarded because they can be implicitly linearized and included into conduction terms. The nodes 7 and 9 corresponding to radiators directly exchange heat with the space.

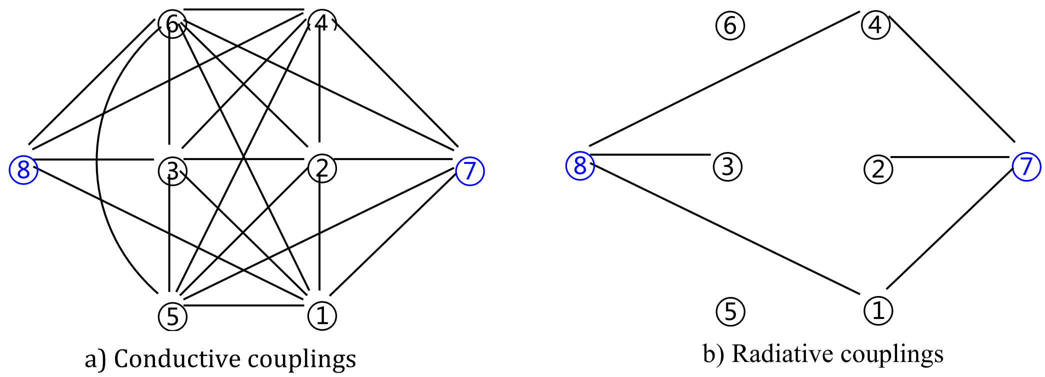

Figure 7. Thermal couplings between nodes

Parameter estimation

The radiative coefficients from radiators to space are directly calculated from their areas and infrared emissivity as mentioned above, while other parameters are to be estimated. There are 22 linear conductances, but seven of them are not estimated but calculated by solving Eq. (17). Six radiative conductances are set only between nodes that could occur large differences in temperatures. The thermal capacity of the node representing IMU is calculated by Eq. (19). After all, only 28 parameters should be estimated.

The searching area of optimization algorithms is decided on the basis of upper and lower bounds of each parameter calculated roughly. The steady state analysis under the average loads (shown in Table 2) is used to establish the equality constraints represented by Eq. (17). At the beginning, the RCGA is used to estimate parameters. If the decrease rate of the WMSE with respect to the generation becomes lower than 10^{-6}°C , the GA procedure is terminated, when the WMSE of the optimal individual is 0.143209℃ and the average WMSE of the whole population is 0.189637℃. The SQP algorithm is continued until the difference of the WMSE between iterations becomes lower than 10^{-6}°C. Then the WMSE is 0.085041℃. Figs. 8 and 9 show the optimization process by the RCGA and the SQP respectively.

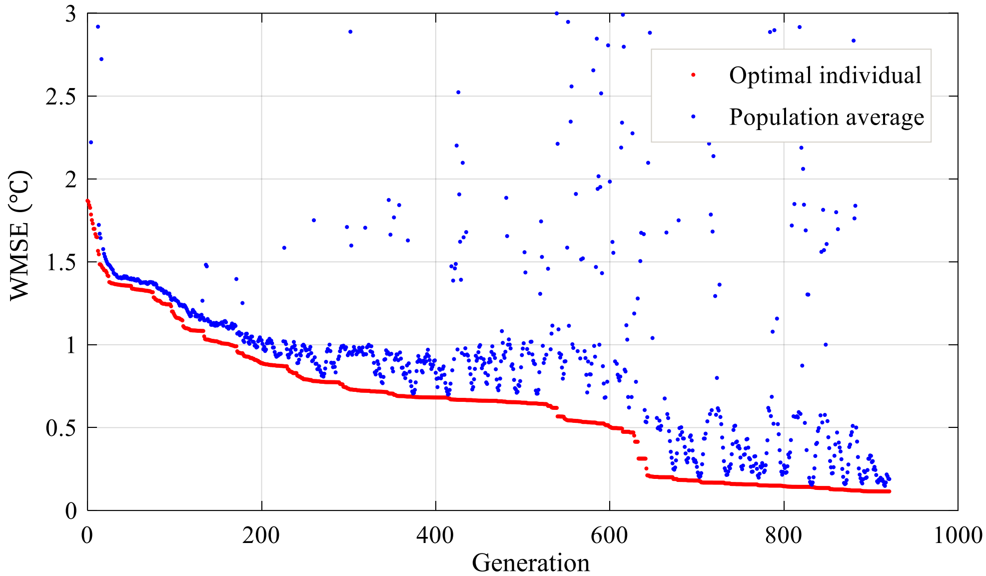

Figure 8. Parameter estimation by RCGA. The value of the penalty function is excluded

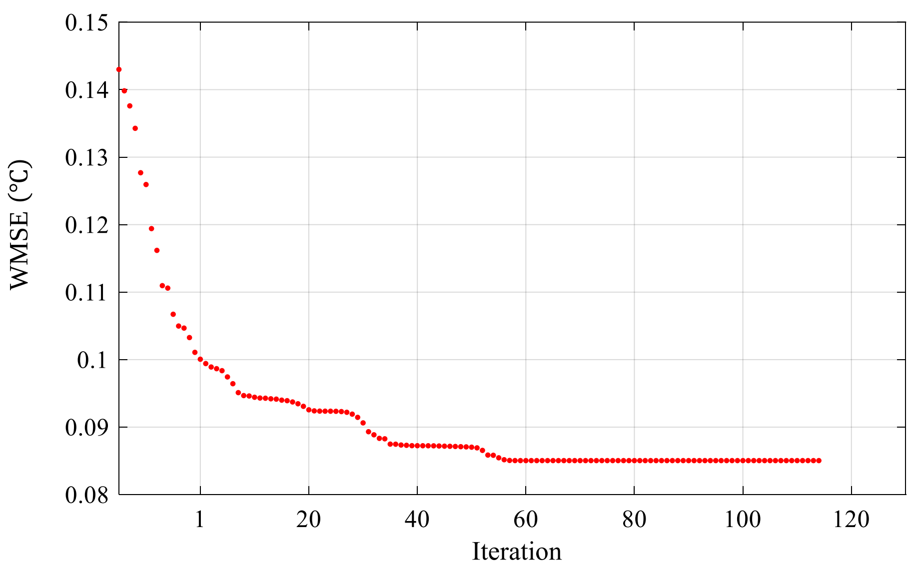

Figure 9. Parameter estimation by SQP

In the early search, the local convergence is prevented by using GA. Judging the degradation of the GA and switching to the SQP, we could improve the convergence rate and computational efficiency. Figure 10 gives the comparison between the temperatures calculated by the reduced model, which is plotted by solid lines, and the initial data, which is plotted by hollow triangles. The sampling period is 60s . Temperatures obtained by the reduced model coincide well with those obtained by the detailed model. Using the periodic solution instead of the long transient solution, the computation cost is saved significantly. The reduced model has a small number of nodes, but its accuracy is almost same with the detailed model.

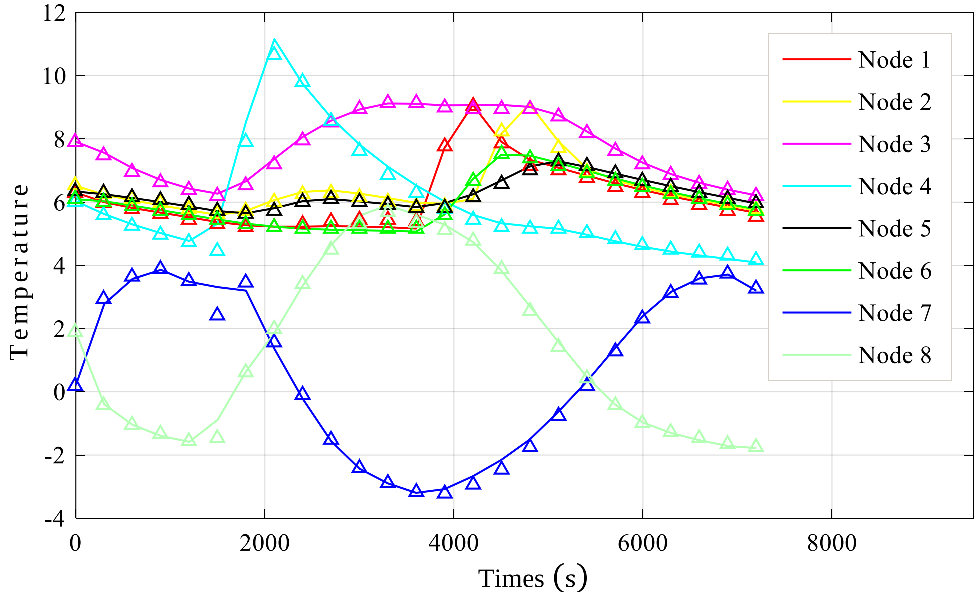

Figure 10. Steady periodic solution of detailed and reduced model

To validate the reduced model, the initial temperature of the whole satellite is assumed to be 10℃, then detailed and reduced models are respectively solved under the periodic loads. The results are compared at nodes 1 and 7. As shown in Figure 11, the solutions of two models properly coincide with each other. Though parameters are estimated by using steady periodic solution, the reduced model represents the behavior in the transient stage with any initial condition successfully. Furthermore, the temperature level and profile in the steady stage have agreed with the solution of the detailed model. This agreement is assured by the linear equality constraints in Eqs. 17 and 19.

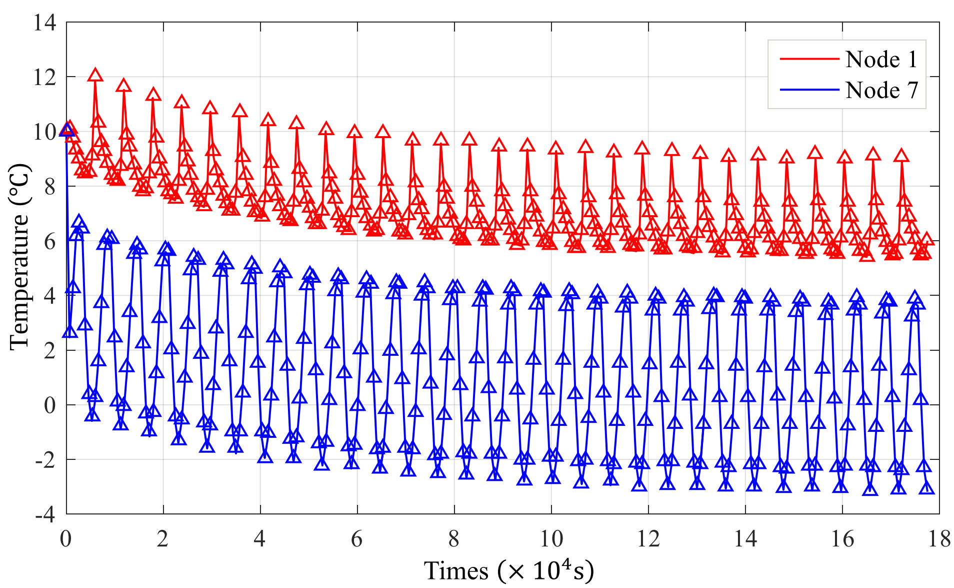

Figure 11. Transient solution of detailed and reduced model

CONCLUSIONS

An approach to estimate parameters of the reduced model by using the periodic solution of a detailed model has been presented in this paper. In order to prepare input data, a simple method to obtain the approximate periodic solution of thermal network equations has been proposed. The long-term behavior of the solution under periodic loads can be approximated by the solution under the average loads. In particular, the average temperatures in the steady periodic solution are constant and have small errors with the equilibrium temperatures under the average loads. Therefore, these temperatures have been used as an initial condition for the transient analysis. In addition, regulating of the phase of loads can accelerate the convergence to the periodic solution. The suitable phase can be determined by the one-nodal analysis. Then, the approximate periodic solution has been obtained by evolving the detailed model during only a few periods.

In the parameter estimation, linear equality constraints using the sum of nodal capacities and the steady state solution under the average loads have been applied. As a result, the identified model can successfully simulate not only the average level of the steady stage temperatures, but also the transient stage with any initial condition. The RCGA and SQP have been cooperated to improve both the global and local search capabilities. The constraints mentioned above have also been used to reduce the dimension of the search space of RCGA to improve search efficiency.

Applying this approach to the thermal modeling of a microsatellite, we have obtained the simple thermal network model which properly represents both transient and steady state behaviors. This approach is applicable to thermal network modeling of more complex satellites, when it will become more apparent that computational cost is reduced and search capability is enhanced by using of the periodic solution and several constraints.

ACKNOWLEDGMENT

The authors would like to thanks Prof. Dr. Yun Chol and Kim Kyong Sok who gave valuable guidance to the writing of the paper.

FUNDING

This work was partially supported by **Kim Il Sung** University.

CONFLICTS OF INTEREST

The authors declare no conflicts of interest.

REFERENCES

[1] D.G. Gilmore, Spacecraft Thermal Control Handbook. Fundamental Technologies, second ed., Vol. 1, The Aerospace Press, El Segundo, California, 2002. http://dx.doi.org/10.2514/4.989117

[2] M. Deiml, M. Suderland, P. Reiss, M. Czupalla, Development and evaluation of thermal model reduction algorithms for spacecraft, Acta Astronaut. 110 (2015) 168–179. https://doi.org/10.1016/j.actaastro.2015.01.018

[3] M. Broussely, Y. Bertin, P. Lagonotte, Reduction and optimisation of thermal models using Kirchhoff network theory, Int. J. Therm. Sci. 42 (2003) 795–804. http://doi.org/10.1016/S1290-0729(03)00052-8

[4] M. Milman, W. Petrick, A note on the solution to a common thermal network problem encountered in heat-transfer analysis of spacecraft, Appl. Math.

Model. 24 (2000) 861–879. https://doi.org/10.1016/S0307-904X(00)00021-4

[5] J. Gaite, Nonlinear analysis of spacecraft thermal models, Nonlinear Dynam. 65 (3) (2010) 283-300. http://dx.doi.org/10.1007/s11071-010-9890-4

[6] I. Pérez-Grande, A. Sanz-Andrés, C. Guerra, G. Alonso, Analytical study of the thermal behaviour and stability of a small satellite, Appl. Therm. Eng. 29 (2009) 2567–2573. http://dx.doi.org/10.1016/j.applthermaleng.2008.12.038

[7] J. Gaite, G. Fernandez-Rico, Linear approach to the orbiting spacecraft thermal problem, J. Thermophys. Heat Transfer 26 (2012) 511–522. http://dx.doi.org/10.2514/1.T3748

[8] C. Arduini, G. Laneve, S. Folco, Linearized techniques for solving the inverse problem in the satellite thermal control, Acta Astronaut. 43 (1998) 473–479. http://dx.doi.org/10.1016/S0094-5765(98)00180-5

[9] T. Akita, R. Takaki, E. Shima, A new adaptive estimation method of spacecraft thermal mathematical model with an ensemble Kalman filter, Acta Astronaut. 73 (2012) 144–155. https://doi.org/10.1016/j.actaastro.2011.11.003

[10] I. Torralbo, I. Perez-Grande, A. Sanz-Andres, J. Pigueras, Correlation of spacecraft thermal mathematical models to reference data, Acta Astronaut. 144 (2018) 305-319. https://doi.org/10.1016/j.actaastro.2017.12.033

[11] E. Anglada, I. Garmendia, Correlation of thermal mathematical models for thermal control of space vehicles by means of genetic algorithms, Acta Astronaut. 108 (2015) 1–17. https://doi.org/10.1016/j.actaastro.2014.11.042

[12] T. Beck, A. Bieler, N. Thomas, Numerical thermal mathematical model correlation to thermal balance test using adaptive particle swarm optimization (APSO), Appl. Therm. Eng. 38 (2012) 168–174. https://doi.org/10.1016/j.applthermaleng.2012.01.027

[13] I. Garmendia, E. Anglada, Thermal mathematical model correlation through genetic algorithms of an experiment conducted on board the International Space Station, Acta Astronaut. 122 (2016) 63–75. https://doi.org/10.1016/j.actaastro.2016.01.022

[14] E. Anglada, L. Martinez-Jimenez, I. Garmendia, Performance of gradient-based solutions versus genetic algorithms in the correlation of thermal mathematical models of spacecrafts, Int. J. Aerosp. Eng. 1-12, 2017. https://doi.org/10.1155/2017/7683457

[15] C. K. Krishnaprakas, Application of accelerated iterative methods for solution of thermal models of spacecraft, J. Spacecraft Rockets, 32 (4) (1995) 608-611. http://dx.doi.org/10.2514/3.26660

[16] C. K. Krishnaprakas, Efficient solution of spacecraft thermal models using preconditioned conjugate gradient methods, J. Spacecraft Rockets, 35 (6) (1998) 760-764. http://dx.doi.org/10.2514/2.3413

[17] J. Nocedal, S. J. Wright, Numerical Optimization, 2nd ed., Springer, 2006. https://www.springer.com/gp/book/9780387303031