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Study on the extension of feasible design region and optimum for effective engineering elastic moduli of composite laminate by using continuous variables

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Abstract

In this paper, the feasible region of design variables is extended by using continuous variables, and an optimization method to find the optimum satisfying various design requirements on in-plane and bending effective engineering moduli is proposed with the genetic algorithm (GA). In the optimal design of effective engineering moduli, we treat the layer orientation angle as the continuous variables, not as the discrete variables such as 0°, ±45° and 90°, and extend the feasible solution region to find the better optimal solutions than those in the case of the limited design variable region. Then, the feasible design region is extended to 1.23•1027, 1.41•1024 times in comparison with the case of the discrete variables such as {0°, ±45°, 90°}and {0°, ±30°, ±45°, ±60°, 90°}, and the optimum that satisfies the severer design requirement and whose effective engineering moduli increase by about 4.47% of those of the discrete variables can be found. And the methods of various maximum moduli design and the criterion moduli design for requirements on in-plane and bending effective engineering moduli that arise from application are proposed. In order to solve these optimization problems we selected the reasonable genetic operators of GA to increase the global property and convergence of solutions.The GA operation with the proposed genetic operators increases the variety of individual population to rise about 2.7 times the convergence rate, and to rise the average fitness of the optimum by 6.7%. The effectiveness of the proposed method is demonstrated by the numerical examples.

Keywords: *Composite laminate, optimization design, effective engineering moduli, genetic algorithm*

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1. Introduction

Because fiber reinforced composite materials have the high specific strength and specific stiffness, they are noted as ideal materials to be widely used in the structures for aerospace in which the lightweight property is needed.

An important problem in the application of composite is to establish the reliable methods of material design and structural design satisfying the design requirements on the given mechanical properties.

Many design methods to find the stacking sequence satisfying of the design requirements desired on various mechanical properties of laminated composites including strength and stiffness have been studied and presented [1-4, 8-10, 17, 18].

Specifically, many optimization design methods on stiffness of laminated composites have been studied to be applied [4, 10].

Composite materials for the aircraft constructions must be designed to exhibit the sufficient resistance to tension deformation, bending deformation, buckling deformation and deflection [8, 10].

Generally, optimization problem on stiffness of laminated composites comes to an optimization problem to find the multi-extreme solution defining the stacking configuration and sequence with the desired stiffness properties.

For these optimization problem the analytical method and heuristic method have been used, because in the analytical method, the gradient informations of objective function and constraints can be used and fall into the local extremes, the excellent heuristic methods such as GA and simulated annealing method(SA) that can obtain the global optimum are being used [5, 6, 10].

Various modified GAs and SAs have been used in the maximum strength design under different loads, maximum natural frequency design, maximum buckling load design, maximum impact load design, in-plane and buckling stiffness design and minimum weight design of the laminated composites [5-8, 11-16, 19, 20].

In these design methods, the design variables, the orientation angles of fiber in each layer are limited to some special angles such as 0°, ±45° , 90° and etc. with the consideration of engineering problems in manufacture to simplify the design problems.

But, because the feasible region of design variables is restricted in these methods, the accuracy of optimal solutions is low and the solution may not be presented at all according to the design requirement.

With the development of manufacturing technology for the composite constructions, the lamination handling has been automated and semi-automated in the winding constructions so that the previous limit of manufacture engineering has been removed and the lamination handling by an arbitrary orientation angle can be realized.

Therefore, if the design variables, the orientation angles of each layer are treated as the continuous variables in the laminate design, the more optimal solutions may be found, the number of feasible solutions be increased and the economic effectiveness be raised.

Although the design methods in which the orientation angles of each layer are treated as the continuous variables, not as the discrete variables have been proposed, those have not be applied in the optimization of various effective engineering moduli [6, 7].

In the practical optimization of effective engineering moduli, the maximization problems for the given elastic moduli as well as the optimization for those criterions are included according to various load conditions.

In these optimization problems the reasonable genetic strategy of GA should be made and its genetic opreators be selected for the given problems in order to raise the global property and convergence of solutions [5, 19].

In this paper the design variables, the ply orientation angles are extended to the continuous variable space so that those could be treated as the arbitrary angles, not as the some special angles for the optimization of effective engineering moduli in the laminate, and a method to solve the problem is proposed with GA.

Specifically, in this paper the optimization of effective engineering moduli of the laminate is classified into the maximum moduli design and the criterion moduli design, and optimizations for various forms of moduli are implemented.

Furthermore, the genetic strategy and genetic operators are reasonably selected in order to raise the global property and convergence of solutions in use of GA for these optimizations, and the effectiveness of the proposed method is demonstrated with the calculation examples.

2. Problem Formulation on the optimization for in-plane and bending moduli

2.1. Constitution equation and stiffness property for fiber reinforced laminates

In the classical laminated plate theory [4, 7], the constitution equation for a laminated plate can be written as follows;

$$\begin{Bmatrix} \mathbf{N} \\ \mathbf{M} \end{Bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{B} & \mathbf{D} \end{bmatrix} \begin{Bmatrix} \boldsymbol{\varepsilon}_0 \\ \mathbf{k} \end{Bmatrix}$$

(1)

where N and M are the stress and moment resultants, respectively; ε_0, k are the strains and the curvatures at the mid-plane, respectively; and A, B, D are the in-plane, coupling and out-of-plane stiffness, respectively.

Finding the effective engineering moduli of a laminated plate from the stiffness matrix components, those can be written as [10]:

$$E_x = \frac{|\mathbf{A}|}{(A_{22}A_{66} - A_{26}^2)t}, \quad E_Y = \frac{|\mathbf{A}|}{(A_{11}A_{66} - A_{16}^2)t}$$
$$G_{xy} = \frac{|\mathbf{A}|}{(A_{11}A_{22} - A_{12}^2)t}, \quad \nu_{xy} = \frac{A_{12}A_{66} - A_{16}A_{26}}{(A_{22}A_{66} - A_{26}^2)}$$
$$E_x^f = \frac{12|\mathbf{D}|}{(D_{22}D_{66} - D_{26}^2)t^3}, \quad E_Y^f = \frac{12|\mathbf{D}|}{(D_{11}D_{66} - D_{16}^2)t^3}$$
$$G_{xy}^f = \frac{12|\mathbf{D}|}{(D_{11}D_{22} - D_{12}^2)t^3}, \quad \nu_{xy}^f = \frac{D_{12}D_{66} - D_{16}D_{26}}{(D_{22}D_{66} - D_{26}^2)}$$
(2)

where E_x, E_y, G_{xy}, ν_{xy} are in-plane effective engineering modulus in longitudinal direction, in-plane effective engineering modulus in transverse direction, in-plane effective engineering shear modulus, in-plane effective engineering Poisson ratio, respectively, and $E_x^f, E_y^f, G_{xy}^f, \nu_{xy}^f$ are bending effective engineering modulus in longitudinal direction, bending effective engineering modulus in transverse direction, bending effective engineering **shear modulus, bending effective engineering Poisson ratio, respectively.**

2.2. Optimization models on in-plane and bending effective engineering moduli

Optimization design on effective engineering moduli presented in practice can be classified into the maximum moduli design and the criterion moduli design.

A general design method in which the optimization for the criterions of moduli and the maximization for moduli can be realized, has not been proposed yet [17, 18].

Dividing the optimization design on effective engineering moduli into the maximum moduli design and the criterion moduli design, we have implemented optimizations for various forms of moduli.

The purpose of the maximum moduli design is to maximize the given specific moduli when the effective engineering moduli of laminate satisfy the design constraints (inequality constraints), and one of the criterion moduli design is to make to achieve the moduli of laminate to the chosen criterion.

2.2.1. Model of maximum moduli design

When the stacking configuration of composite laminate is expressed as $[(\theta_1)_{N1}/(\theta_2)_{N2}/\ldots/(\theta_i)_{Ni}/\ldots/(\theta_n)_{Nn}]_S$, the maximum moduli design is a typical muti-objective optimization problem and it can be mathematically formulated as follows.

In case of maximum in-plane effective engineering moduli design

find: $X=(\theta_1, \ldots, \theta_n, N_1, \ldots N_n)^T$

$$\max_{\mathbf{X}} : F_{\max}^{in}\left(E_x(\mathbf{X}), E_y(\mathbf{X}), G_{xy}(\mathbf{X})\right)$$

$$\text{such that}: \begin{cases} -90° \le \theta_i \le 90°, (i=1,\cdots n) \\ N_i \le 4, \ (i=1,\cdots n) \\ E_x(\mathbf{X}) \ge E_x^C, \ E_y(\mathbf{X}) \ge E_y^C, G_{xy}(\mathbf{X}) \ge G_{xy}^C \\ E_x^f(\mathbf{X}) \ge E_x^{fC}, \ E_y^f(\mathbf{X}) \ge E_y^{fC}, G_{xy}^f(\mathbf{X}) \ge G_{xy}^{fC} \end{cases}$$
(3)

where E_x^C, E_y^C, G_{xy}^C are the critical values of in-plane effective engineering moduli and E_x^{fC}, E_y^{fC}, G_{xy}^{fC} those of bending effective engineering moduli.

In the case of maximum bending effective engineering moduli design

find: X=(θ 1, …, θ n, N1, …Nn)T

$$\max_{\mathbf{X}} : F_{\max}^{f}\left(E_x^f(\mathbf{X}), E_y^f(\mathbf{X}), G_{xy}^f(\mathbf{X})\right)$$

$$\text{such that}: \begin{cases} -90° \le \theta_i \le 90°, (i=1,\cdots n) \\ N_i \le 4,\ (i=1,\cdots n) \\ E_x(\mathbf{X}) \ge E_x^C,\ E_y(\mathbf{X}) \ge E_y^C, G_{xy}(\mathbf{X}) \ge G_{xy}^C \\ E_x^f(\mathbf{X}) \ge E_x^{fC},\ E_y^f(\mathbf{X}) \ge E_y^{fC}, G_{xy}^f(\mathbf{X}) \ge G_{xy}^{fC} \end{cases} \tag{4}$$

In the case of maximum in-plane and bending effective engineering moduli design

find: X=(θ 1, …, θ n, N1, …Nn)T

$$\max_{\mathbf{X}} : F_{\max}^{in-f}\left(E_x(\mathbf{X}), E_y(\mathbf{X}), G_{xy}(\mathbf{X}), E_x^f(\mathbf{X}), E_y^f(\mathbf{X}), G_{xy}^f(\mathbf{X})\right)$$

$$\text{such that}: \begin{cases} -90° \le \theta_i \le 90°, (i=1,\cdots n) \\ N_i \le 4,\ (i=1,\cdots n) \\ E_x(\mathbf{X}) \ge E_x^C,\ E_y(\mathbf{X}) \ge E_y^C, G_{xy}(\mathbf{X}) \ge G_{xy}^C \\ E_x^f(\mathbf{X}) \ge E_x^{fC},\ E_y^f(\mathbf{X}) \ge E_y^{fC}, G_{xy}^f(\mathbf{X}) \ge G_{xy}^{fC} \end{cases} \tag{5}$$

Because the maximum moduli design of composite laminate is a multi-objective optimization problem, by using the weight coefficient method the objective functions of Eq.(3)-Eq.(5) can be converted into the single objective functions as follows.

$$F_{\max}^{in}(\mathbf{X}) = \alpha_x \frac{E_x(\mathbf{X}) - E_{x(\min)}}{E_{x(\max)} - E_{x(\min)}} + \alpha_y \frac{E_y(\mathbf{X}) - E_{y(\min)}}{E_{y(\max)} - E_{y(\min)}} + \alpha_{xy} \frac{G_{xy}(\mathbf{X}) - G_{xy(\min)}}{G_{xy(\max)} - G_{xy(\min)}} \to \max \tag{6}$$

$$F_{\max}^{f}(\mathbf{X}) = \alpha'_x \frac{E_x^f(\mathbf{X}) - E_{x(\min)}^f}{E_{x(\max)}^f - E_{x(\min)}^f} + \alpha'_y \frac{E_y^f(\mathbf{X}) - E_{y(\min)}^f}{E_{y(\max)}^f - E_{y(\min)}^f} + \alpha'_{xy} \frac{G_{xy}^f(\mathbf{X}) - G_{xy(\min)}^f}{G_{xy(\max)}^f - G_{xy(\min)}^f} \to \max \tag{7}$$

$$F_{\max}^{in-f}(\mathbf{X}) = \sum_{i=1,2,6} \left(\alpha_i \frac{E_i(\mathbf{X}) - E_{i(\min)}}{E_{i(\max)} - E_{i(\min)}} + \alpha'_i \frac{E_i^f(\mathbf{X}) - E_{i(\min)}^f}{E_{i(\max)}^f - E_{i(\min)}^f} \right) \to \max \tag{8}$$

Where $E_{i(\min)}$, $E_{i(\max)}$ (*i*= *x*, *y*, *xy*) are the maximum and minimum of in-plane effective engineering moduli, respectively, $E_{i(\min)}^f$, $E_{i(\max)}^f$ (*i*= *x*, *y*, *xy*) are the maximum and minimum of bending effective engineering moduli, respectively, and α_i, α'_i (*i*= *x*, *y*, *xy*) are the weight coefficients of components of in-plane and bending effective engineering moduli, respectively.

2.2.2. Model of criterion moduli design

The criterion moduli design is a multi-objective optimization problem that determines the stacking configuration and sequence of laminate so that its effective moduli reach to the given design criterion, and can be formulated by using the weight coefficient method such as 2.2.1.

For the above stacking configrutaion, if the desired criterion values of effective engineering moduli are represented by

E_i^*, E_i^{f*} (i=x, y, xy), models of the criterion moduli design can be formulated as follows.

In the case of in-plane criterion moduli design

find: X=(θ_1, …, θ_n, N_1, …N_n)^T

$$\max_{\mathbf{X}} : F_C^{in}(\mathbf{X}) = 1 - \sum_{i=x,y,xy} \left[\alpha_i \left(\frac{E_i(\mathbf{X}) - E_i^*}{E_i^*} \right)^2 \right]$$

(9)

$$\text{such that} : \begin{cases} -90° \le \theta_i \le 90°, (i = 1, \cdots n) \\ N_i \le 4, \ (i = 1, \cdots n) \\ E_x^f(\mathbf{X}) \ge E_x^{fC}, \ E_y^f(\mathbf{X}) \ge E_y^{fC}, G_{xy}^f(\mathbf{X}) \ge G_{xy}^{fC} \end{cases}$$

where α_i (i= x, y, xy) is the weight coefficients of components of in-plane effective engineering moduli.

In the case of bending criterion moduli design

find: X=(θ_1, …, θ_n, N_1, …N_n)^T

$$\max_{\mathbf{X}} : F_C^f(\mathbf{X}) = 1 - \sum_{i=x,y,xy} \left[\alpha_i' \left(\frac{E_i^f(\mathbf{X}) - E_i^{f*}}{E_i^{f*}} \right)^2 \right]$$

(10)

$$\text{such that} : \begin{cases} -90° \le \theta_i \le 90°, (i = 1, \cdots n) \\ N_i \le 4, \ (i = 1, \cdots n) \\ E_x(\mathbf{X}) \ge E_x^C, \ E_y(\mathbf{X}) \ge E_y^C, G_{xy}(\mathbf{X}) \ge G_{xy}^C \end{cases}$$

where α_i' (i= x, y, xy) is the weight coefficients of components of bending effective engineering moduli.

In the case of in-plane and bending criterion moduli design

find: X=(θ_1, …, θ_n, N_1, …N_n)^T

$$\max_{\mathbf{X}} : F_C^{in-f}(\mathbf{X}) = 1 - \sum_{i=x,y,xy} \left[\alpha_i \left(\frac{E_i(\mathbf{X}) - E_i^*}{E_i^{0*}} \right)^2 + \alpha_i' \left(\frac{E_i^f(\mathbf{X}) - E_i^{f*}}{E_i^{f*}} \right)^2 \right]$$

(11)

$$\text{such that} : \begin{cases} -90° \le \theta_i \le 90°, (i = 1, \cdots n) \\ N_i \le 4, \ (i = 1, \cdots n) \end{cases}$$

3. Optimization for effective engineering moduli with genetic algorithm

GA is an optimization technique based on the principle of natural evolution that the fittest individuals are survived, and are widely used in the composite design.

Here, an important problem is to select the suitable genetic strategy and operators for the given optimization problem.

3.1. Feasible region of design variables

In the design of laminate the design variables are the fiber orientation angle and ply numbers of each ply group.

In design of laminated composite materials, if the orientation angles of each layer is limited to the discrete variables such as 0° , ±45° and 90°, because of restriction in the feasible region of design variables the solutions for the design requirements could exist little or not exist at all in the extreme cases.

If the fiber orientation angles are extended to the continuous variable space, the accuracy and global property of the optimal solutions on the given design condition can be increased.

Therefore, the fiber orientation angle is treated as the continuous variable and the ply number of each ply group into the integer from 1 to 4.

When the stacking configuration of composite laminate is expressed as $[(\theta_1)_{N1}/(\theta_2)_{N2}/.../(\theta_i)_{Ni}/.../(\theta_n)_{Nn}]_S$, the feasible region of design variables is defined as

$$\theta_i \in [-90, 90], (i=1,\cdots n)$$
$$N_i \in \{1, 2, 3, 4\}, \ (i=1,\cdots n)$$

(12)

3.2. Fitness function and treatment of constraint

Fitness function and treatment of constraint have important effect on the accuracy, convergence and global property of the solutions.

If the individuals very near to the feasible region that are very excellent in the objective function but deviated lightly from the constraint are created, it provides the favourable condition for the variety and convergence of solutions, and those can be satisfied with the constraint with the later operations.

But, if the constraint is mistaken so that these individuals are eliminatd, it have bad effect on the optimization search of GA so that the solution can fall into a local extreme or the convergence not be ensured.

The penalty method used frequently in the treatment of constraint, adds the suitable penalty term to the objective function when the solution violates the constraint so that the operation is proceeded in the direction with satisfying constraint and improving of objective function value [15].

Therefore, it is very important to add the penalty term on the constraint according to the given optimization problem in GA.

For n constraints on effective engineering moduli, $E_i(\mathbf{X}) \geq E_i^c, (i=1,\cdots n)$, adding the penalty terms to the objective functions F(X) of Eq.(6)~Eq.(11) the fitness function can be defined as follows;

$$F^C(\mathbf{X}) = \begin{cases} F(\mathbf{X}), & E_i(\mathbf{X}) \geq E_i^C, (i=1,\cdots,n) \\ F(\mathbf{X}) \cdot \prod_i \left(\dfrac{E_i(\mathbf{X})}{E_i^C} \right), & E_i(\mathbf{X}) < E_i^C, (i=1,\cdots,m, \ m \leq n) \end{cases}$$

(13)

3.3. Generation of initial population

With the consideration of global property, accuracy and convergence of solutions, gene is encoded by the binary stringer.

Then, because the continuous variable, the fiber orientation angle is in the region of -90°≦ θ_i ≦90°, when its accuracy is selected by 0.1°, $2^{10} < 1800 < 2^{11}$, its gene corresponds to 11 bits string.

And, because the discrete variable, maximum ply number of each ply group is 4, it can be encoded by 2 bits string.

Therefore, an individual chromosome is expressed by a string of 13N bits in case of the laminate consisting of N layers.

In the feasible region design variables, 200 individuals are randomly generated to form an initial population.

3.4. Genetic strategy and genetic operation mechanism

Because the optimization for effective engineering moduli of laminate is a multi-objective optimization problem with multi-extremes, with the consideration of global property, accuracy and convergence of the solution, a suitable genetic strategy must be established and genetic operation mechanism corresponding to it be chosen.

The genetic strategy for this problem is that the dominant individuals are preserved to ensure the accuracy and convergence of solution, and that the variety of individual population is raised to remove the riskiness that the solutions fall into the local extremes and to obtain the global optimal solutions.

According to this genetic strategy, the genetic operators must be rationally chosen.

3.4.1. Selection operator

In the paper, Multiple Elitist that some individuals with high fitness are preserved to the offspring generation to accelerate the creation of the dominant individuals is used.

The dominant individuals with high fitness of 5~20% in parent generation are preserved to the next generation to increase the creation probability of individuals with higher fitness in the population, so that the accuracy and convergence rate of solution are raised.

The selection operator is chosen by Roulette Wheel mechanism.

3.4.2. Crossover operator

The more similar the gene strings of two parent individuals selected by the mating pool in crossover, i.e., the nearer two designs each other, the lower the variety of population and the greater the riskiness that the solutions fall into the local extremes.

Therefore, in order to prevent the convergence phenomenon to the local extremes with inbreeding and to raise the variety of population and the global property of solution in the crossover stage, the crossover operator with Hamming distance is implemented.

A basic idea of this crossover mechanism is that, two individuals which have same genes over 70% in those gene strings are assumed to be near each other so that the crossover operation is not implemented, and after mutating one with the low fitness so that those Hamming distance is satisfied, the crossover operation is implemented.

In order to raise the convergence rate and the global property, even the crossover operation is implemented with Scattered crossover, not with single point or two-point crossover [15].

Then, the crossover probability is chosen by 0.8.

3.4.3. Mutation operator

In order to raise the variety of population and the global property of solution, with choosing randomly of mutating individuals, the number of mutating genes and even the mutating points, the mutation operation is implemented. (This operator is called as Random mutation.) Then, the mutation probability is chosen by 0.01.

The generated individuals are included in the population to form a new population, and, after estimating of their fitness the genetic operation is repeated until the convergence criterion is satisfied.

The GA flowchart for effective engineering moduli optimization of laminate is shown in Fig.1.

4. Numerical experiments and discussion

Through the numerical experiments the effectiveness of the optimization method for effective engineering moduli of laminate proposed in this paper is demonstrated.

Taking Graphite fiber/epoxy resin (T300/5208) laminate for example, the numerical experiment is proceeded and its basic property is given in Table 1 [20]. Then, the number of total layers in laminate is chosen by N=10.

First, taking maximum design on in-plane effective engineering moduli of Table 2 as an example, results in the case that fiber angles are fixed by the special angles- {0º, ±45º, 90º} and {0º, ±30º, ±45º, ±60º, 90º} such as previous methods are compared with those in the case that those are chosen by the continuous variable.

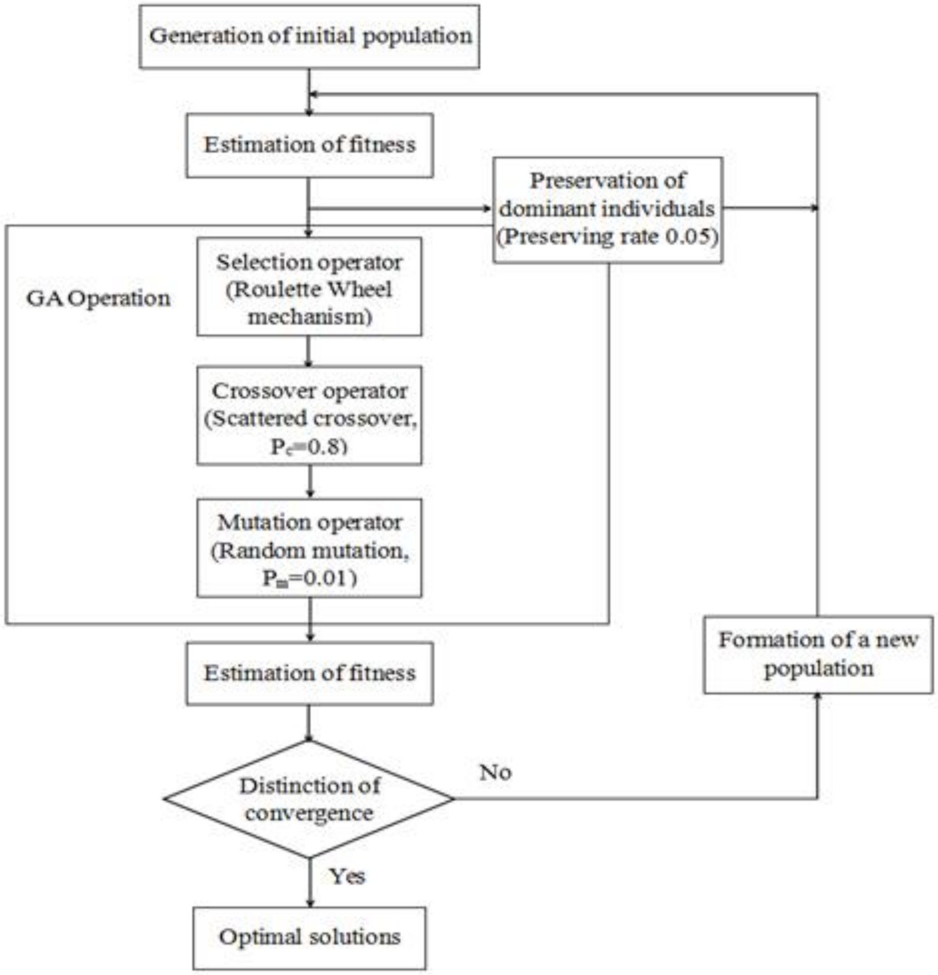


Figure 1. *GA flowchart for effective engineering moduli optimization of laminate*

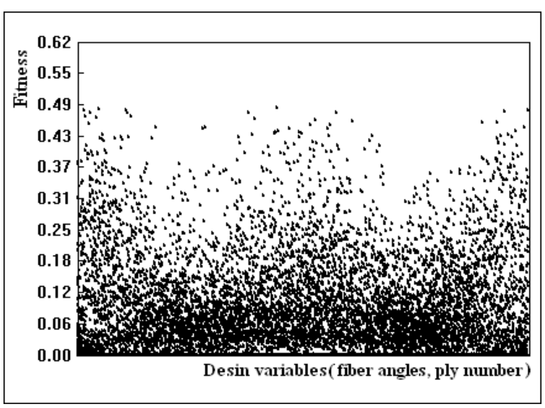
Table1. *Basic property of T300/5208 unidirectional fiber-reinforced composites*

Property	**Value**
Modulus in longitudinal direction, E_1	181.0 GPa
Modulus in transverse direction, E_2	10.3 GPa
Shear modulus, G_{12}	7.17 GPa
Poisson's ratio, ν_{12}	0.28
Density, ρ	1.6 g/cm^3

Table2. *Condition of maximum design on in-plane effective engineering moduli*

Methods	**Method1**	**Method2**	**Method3**
Material	T300/5208	T300/5208	T300/5208
Design variables	θi- discrete variable $\theta i \in \{0°, \pm 45°, 90°\}$ $N_i \in \{1, 2, 3, 4\}$ (i=1, …, n)	θi - discrete variable $\theta i \in \{0°, \pm 30°, \pm 45°, \pm 60°, 90°\}$ $N_i \in \{1, 2, 3, 4\}$	θi- continuous variable $\theta i \in [-90°, 90°]$ $N_i \in \{1, 2, 3, 4\}$ (i=1, …, n)
Constraints	$E_x(\mathbf{X}) \geq 60\,\text{GPa}$, $E_x^f(\mathbf{X}) \geq 60\,\text{GPa}$, $E_y(\mathbf{X}) \geq 50\,\text{GPa}$, $E_y^f(\mathbf{X}) \geq 60\,\text{GPa}$, $G_{xy}(\mathbf{X}) \geq 10\,\text{GPa}$, $G_{xy}^f(\mathbf{X}) \geq 12\,\text{GPa}$		
Design objective	$E_x(\mathbf{X}) \to \max$, $E_y(\mathbf{X}) \to \max$, $G_{xy}(\mathbf{X}) \to \max$		

In Figure.2 the distributions of feasible solutions in case of discrete variable-{0°, ±45°, 90°} and in case of continuous variable chosen in [-90.0°, 90.0°) with intervals of 0.1° are shown for above design condition.



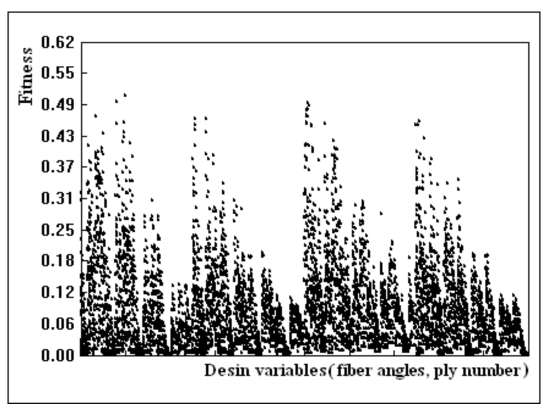


Figure 2. *Distribution of feasible solutions in cases of discrete variable and continuous variable*

a) $\theta_i \in \{0°, \pm 45°, 90°\}$

b) $\theta_i \in [-90.0°, 90.0°)$

The rates of number of feasible solutions in the continuous variable space to one in the discrete variable space are shown in Table 3.

As shown in Table 3, the rates of number of feasible solutions in the case of continuous variable and those in cases of discrete variables-{0°, ±45°, 90°}, {0°, ±30°, ±45°, ±60°, 90°} are $1.23 \cdot 10^{27}$ and $1.41 \cdot 10^{24}$, respectively.

Table 3. *The rates of number of feasible solutions in the continuous variable space to one in the discrete variable space*

Design variable	**Rate of number of feasible solutions**
$\theta_i \in \{0°, \pm45°, 90°\}$	$1.23 \cdot 10^{27}$
$\theta_i \in \{0°, \pm30°, \pm45°, \pm60°, 90°\}$	$1.41 \cdot 10^{24}$

These rates are almostly maintained without regard to the design condition.

This result shows that the feasible solution region is considerably extended in the case of the continuous variable in comparison with the discrete variables.

The comparison of results in the proposed method by the continuous variable with those in the previous method by the discrete variables is shown in Table 4. Their optimal solutions are chosen by 5 solutions with the greatest fitness.

From the result of Table 4, it can be found that, if the fiber angles are treated as the continuous variable, its feasible solution space is extended in comparison with the case of the discrete variables, so that the in-plane effective engineering moduli are increased by about 4.47%, finally, the more optimal solutions can be found.

Table4. *Comparison of results for maximum design on in-plane effective engineering moduli*

Methods	**Optimal solutions**	**In-plane effective engineering moduli (calculated), GPa**							
		E_x	E_y	G_{xy}	ν_{xy}	E_x^f	E_y^f	G_{xy}^f	ν_{xy}^f
Method1	$[45_1/90_1/0_1/-45_2/45_2/0_1/-45_1/90_1]_s$	62. 8	62.8	30.9	0. 4	60.5	65.0	29.9	0.3
	$[0_2/90_1/45_1/-45_1/90_1/0_1/-45_1/45_2/-$	62.	62.8	30.9	0.	104.	61.2	18.3	0.2

	$45_1]_s$	8			4	0			
	$[90_1/0_1/90_1/45_2/-45_2/-45_3/0_1/45_1]_s$	62.8	62.8	30.9	0.4	61.3	96.9	20.0	0.2
	$[0_1/-45_1/90_1/45_1/0_1/-45_1/90_1/-45_1/45_2]_s$	62.8	62.8	30.9	0.4	85.5	60.3	23.2	0.3
	$[90_1/-45_1/45_1/0_2/-45_1/45_2/90_1/-45_1]_s$	62.8	62.8	30.9	0.4	64.7	74.1	27.0	0.3
Method2	$[60_1/90_1/0_3/-45_1/-60_2/60_1/45_1]_s$	69.8	69.8	26.9	0.3	81.4	76.4	18.4	0.2
	$[90_1/-60_1/0_3/60_1/-45_1/45_1/60_1/-60_1]_s$	69.8	69.8	26.9	0.3	80.9	81.6	16.8	0.1
	$[90_1/-45_1/0_2/60_1/0_1/45_1/60_1/-60_2]_s$	69.8	69.8	26.9	0.3	80.4	77.2	20.3	0.2
	$[60_1/90_1/0_2/-45_1/45_1/90_1/-30_1/30_1/-60_1]_s$	69.8	69.8	26.9	0.3	70.9	76.8	19.9	0.2
	$[60_1/90_1/0_2/-45_1/45_1/60_1/-60_1/0_1/-60_1]_s$	69.8	69.8	26.9	0.3	69.8	74.4	20.4	0.2
Method3	$[80.0_1/-5.9_1/-53.0_1/-4.3_2/-60.4_2/55.9_1/52.9_1/36.7_1]_s$	70.5	64.9	28.1	0.3	81.0	75.7	17.2	0.1
	$[-88.1_1/10.1_2/-13.7_1/71.3_1/-56.8_2/46.9_1/49.0_1/-33.8_1]_s$	70.2	64.9	28.2	0.3	92.0	78.3	15.4	0.2
	$[71.6_1/-50.9_1/-6.8_1/29.5_2/39.9_1/-84.8_2/-29.1_2]_s$	69.9	65.6	28.1	0.3	64.8	68.1	27.8	0.3
	$[-11.2_1/5.8_1/84.9_1/-51.6_2/80.2_1/60.1_1/-37.4_1/29.2_2]_s$	69.9	64.9	28.3	0.3	90.3	65.1	18.2	0.2
	$[-89.0_1/5.3_2/65.0_1/-47.6_2/49.3_2/-7.4_1/-62.1_1]_s$	69.4	65.6	28.3	0.3	82.5	81.2	18.6	0.2

When the constraint in the optimization problem of Table 2 is selected by Eq. (14), results of maximum design on in-plane effective engineering moduli are shown in Table 5.

$$\begin{aligned} &E_x^0(\mathbf{X}) \geq 74\,\text{GPa}, && E_x^f(\mathbf{X}) \geq 90\,\text{GPa}, \\ &E_y^0(\mathbf{X}) \geq 80\,\text{GPa}, && E_y^f(\mathbf{X}) \geq 82\,\text{GPa}, \\ &G_{xy}^0(\mathbf{X}) \geq 20\,\text{GPa}, && G_{xy}^f(\mathbf{X}) \geq 14\,\text{GPa} \end{aligned} \qquad (14)$$

Table 5. *Comparison of results for maximum design on in-plane effective engineering moduli*

Methods	Optimal solutions	Objective Function	In-plane effective engineering moduli (calculated), GPa							
			E_x	E_y	G_{xy}	ν_{xy}	E_x^f	E_y^f	G_{xy}^f	ν_{xy}^f
Method1	No solutions	-	-	-	-	-	-	-	-	-
Method2	No solutions	-	-	-	-	-	-	-	-	-
Method3	$[84.0_1/8.7_1/-20.9_1/-85.5_1/13.1_2/69.3_1/-52.2_1/-51.1_1/66.7_1]_s$	0.5145	74.0	80.0	22.4	0.2	91.6	83.8	14.2	0.1
	$[-88.5_1/8.3_2/71.2_1/-24.8_2/-70.4_2/66.5_1/32.0_1]_s$	0.5144	74.0	80.1	22.3	0.2	91.4	83.1	14.8	0.1
	$[-11.1_1/-86.0_2/22.2_2/-1.0_1/72.0_2/-41.4_1/-51.4_1]_s$	0.5141	74.6	80.1	22.1	0.2	91.1	84.5	14.2	0.1
	$[-76.0_1/0.0_2/67.2_1/-6.8_1/73.4_1/74.7_1/27.4_1/-54.2_2]_s$	0.5140	74.2	80.2	22.2	0.2	94.2	82.1	14.2	0.1
	$[-15.5_1/-89.1_2/18.0_3/-48.8_1/71.8_1/77.0_1/-40.0_1]_s$	0.5136	74.1	80.4	22.1	0.2	90.2	82.4	15.6	0.1

From Table 5, it can be found that, in the higher design requirement, the solutions don't exist when the design variables are fixed by the special discrete variables, but the optimal solutions exist when those are the continuous variable.

With the treatment of fiber angles as the the continuous variable, the effectiveness of the proposed genetic strategy and genetic operators is analyzed in comparison with the Simple Genetic Algorithm (SGA). The optimization is assumed as the the maximum design on in-plane effective engineering moduli and the design constraint is selected such as Table 2. First, on the assumption that the size of initial population is 200, the result analyzed the convergence according to the change of number the preserving individuals is shown in Table 6.

Table 6. *Convergence property according to the preserving rate of dominant individuals*

Preserving Number of dominant individuals	Preserving rate of dominant individuals,%	Convergence generation	Average fitness	Average deviation
0	0	440	0.486	0.165
5	2.5	250	0.501	0.134
10	5	120	0.519	0.121
15	7.5	110	0.513	0.124
20	10	70	0.502	0.122

When the size of initial population is assumed to be 200 and the preserving number of dominant individuals to be 10, the change curve of fitness is shown in Fig.3 in comparison with one when the dominant individuals are not preserved.

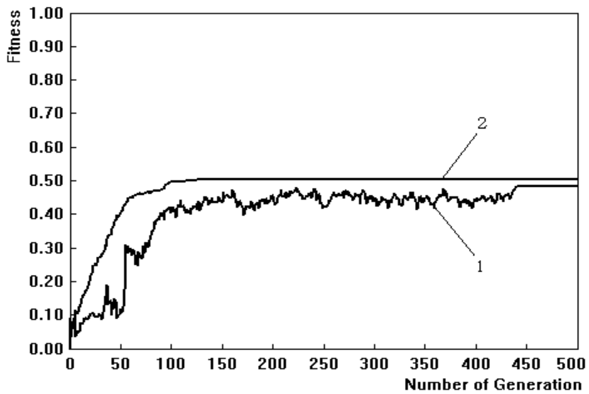


***Figure 3**. Effect of preserving of dominant individuals on convergence.*

1- in case of not preserving of dominant individuals

2- in case of preserving of 10 dominant individuals

From Table 6 and Fig.3, it can be shown that, the convergence rate, average fitness and the deviation of fitness are considerably improved with preserving of dominant individuals to next generation, and the convergence rate is increased but the average fitness slightly decreased when their preserving number is more than 5% of the size of the population.

A result of analyzing the advantage in which the crossover with Hamming distance and Scattered crossover are implemented in crossover operation and Random mutation is used in mutation operation, in comparison with the previous SGA, is shown in Fig.4. Then, the preserving number of dominant individuals is assumed as 10.

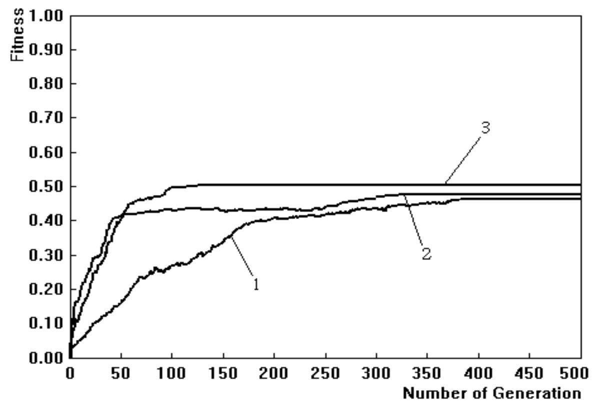


***Figure 4**. Convergence property with various crossover and mutation operators.*

1- single point crossover+simple mutation, 2- two point crossover+simple mutation

3- the proposed genetic operators

From Fig.4, it can be shown that, when the proposed crossover and mutation operators are used, the convergence rate is increased about 2.7 times and the average fitness of optimal solutions is increased by 6.7% in comparison with the previous SGA operators.

These results show that the optimization method with the proposed genetic strategy and genetic operators improves considerably the accuracy and convergence of solutions in comparison with the previous one with SGA.

5. Conclusions

In this paper, it is proved that, when the fiber angles are treated as the continuous variable in the optimization for effective engineering moduli of laminate, its feasible solution region is extended to 1.23•1027, 1.41•1024 times in comparison with the case of the discrete variables such as {0°, ±45°, 90°}, {0°, ±30°, ±45°, ±60°, 90°}, respectively, and the optimum that satisfies the severer design requirement and whose effective engineering moduli are increased by about 4.47% than the case of the discrete variables can be found.

Furthermore, it is found that, in use of GA for the optimization of effective engineering moduli of laminate, the selection operation with the preservation of dominant individuals (preserving rate 0.05) and Roulette wheel mechanism, the crossover operation in which crossover with Hamming distance is combined with Scattered crossover, and Random mutation operation, increase the variety of the individual population to rise about 2.7 times the convergence rate and to rise the average fitness of optimal solutions by 6.7%.

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